dxFeed Theoretical Options Prices, Greeks and Volatilities

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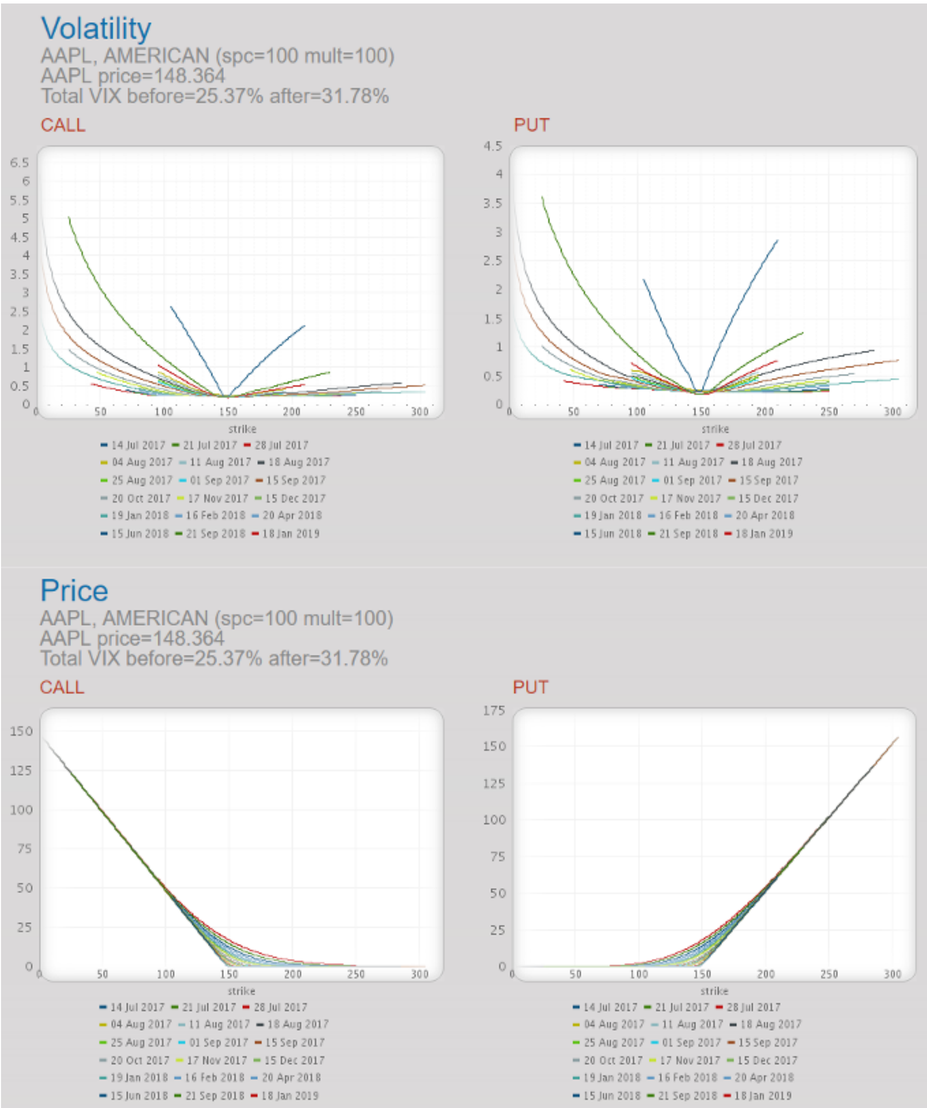
1. dxPrice

dxPrice calculation engine provides arb-free theoretical option prices calculation based on real-time or historical data. Pricing data may be delivered along with real-time data feed via dxFeed API or calculated based on historical onDemand data store.

Option markets often lack liquidity in certain strikes, and the task of determining the market price or a fair settlement price of a certain security is not always trivial. dxPrice is a technology that derives the best possible option prices from whatever market data is available.

Fair prices are required to be as close to market data as possible and to satisfy specific arbitrage-free conditions. The prices are derived from the entire set of bids and offers available for an option series. As a result, they are smooth across the series, arbitrage free, reflect the market and fill in the gaps. It is an important benefit that the algorithm is model neutral and therefore works well on all kinds of asset classes. Cubic splines are used to approximate option's objective prices. The resulting spline curve is finally considered as fair options price curve. We rewrite arbitrage-free conditions as linear constraints on spline's parameters and obtain strictly convex quadratic optimization problem. The latter problem can be solved using Goldfarb-Idnani method.



The technology can be used in a variety of applications such as augmentation of live market data feeds, on-the-fly risk control, end-of-day processing, or market making.

Key benefits:

1. Smooth price and volatility curves.
2. Prices are arbitrage free across the option class.
3. Best fits the market prices available.
4. Works well on low liquidity and high liquidity markets as well as in crash scenarios.
5. Fast enough to be used in real-time.

dxPrice calculates theoretical option prices for an underlying for a given moment in history starting from January 1, 2010 and up to the previous trading day for the current date.

2. Greeks and Implied Volatility

Because the option price does not always appear to move in conjunction with the price of the underlying asset, it is important to understand what factors contribute to the movement in the price of an option, and what effect they have.
Using theoretical prices that we calculate with dxFeed, we provide calculations volatility and of Greeks: **delta**, **gamma**, **vega**, **theta** and **rho** of option positions.

Implied Volatility is a measure of how much the marketplace expects asset price to move for an option price.

In calculation we use adjustments for theoretical prices and greeks according to shifts in underlyings' prices.

Methods used for calculation:

- Black-Scholes
- Binominal Tree (Cox-Ross-Rubenstein)

Greeks event is a snapshot of the option price, Black-Scholes volatility and greeks. It represents the most recent information that is available about the corresponding values on the market at any given moment of time.

2.1. Data coverage

OPRA, ISE Spread Book, CME, CBOT, NYMEX, COMEX, ICE US, ICE Europe, CFE: Cboe Futures Exchange, Nasdaq Futures (NFX), Eurex, Borsa Istanbul

3. The model

European call and put options of the same series and strike satisfy call-put parity equality of the following form: $C - P = \frac{U}{Q(\tau)+1} - \frac{K}{R(\tau)+1}$ where:

- C is the call option price;
- P is the corresponding put option price;
- U is the underlying price;
- K is the strike price;
- $Q(\tau)$ is the simple dividend return during the duration τ of the option;
- $R(\tau)$ is the simple interest (risk-free) return during the duration τ of the option.

Non-standard or adjusted options (when the number of underlying deliverables per contract is different from the option price dollar value multiplier) strike price and/or option prices may be represented in different units that underlying price. For the purposes of the above formulae both strike price, call price, and put price has to be represented in the same units as the underlying price using an appropriate additional multipliers.
Here, the simple dividend return and interest return are related to the annualized continuously compounded dividend yield q and the annualized continuously compounded interest rate r via the following formulae: $Q(\tau) = e^{q\tau} - 1$, $R(\tau) = e^{r\tau} - 1$ where τ is the duration of the option represented in fractions of a year.

Forward price and cost of carry

Forward price of the underlying F can be expressed in the terms of simple returns $Q(\tau)$ and $R(\tau)$:

$$F = \frac{R(\tau)+1}{Q(\tau)+1}U = e^{(r-q)\tau}U = e^{b\tau}U$$

Here b is an annualized continuously compounded cost of carry which relates to dividend yield q and interest rate r via the following formula: $b = r - q$. The discount factor D that ties underlying price to forward prices as $U = DF$ can be expressed as: $D = \frac{Q(\tau)+1}{R(\tau)+1} = e^{-b\tau}$

Futures as underlying

Theoretically futures underlyings have zero cost of carry, so for an options on futures $q = r$ and, correspondingly $Q(\tau) = R(\tau)$ and $U = F$.

Black-Scholes

Black-Scholes formula can be directly expressed in the terms of simple returns $Q(\tau)$ and $R(\tau)$.

$$C = \frac{UN(d_+)}{Q(\tau)+1} - \frac{KN(d_-)}{R(\tau)+1} = e^{-r\tau}(FN(d_+) - KN(d_-))$$

$$P = \frac{KN(-d_-)}{R(\tau)+1} - \frac{UN(-d_+)}{Q(\tau)+1} = e^{-r\tau}(KN(-d_-) - FN(-d_+))$$

$$d_\pm = \frac{1}{\sigma(\tau)} ln[\frac{U(R(\tau)+1)}{K(Q(\tau)+1)}] \pm \frac{1}{2}\sigma(\tau) = \frac{1}{\sigma(\tau)} ln[\frac{F}{K}] \pm \frac{1}{2}\sigma(\tau)$$

where:

- $N(\cdot)$ is the cumulative normal distribution function;
- $\sigma(\tau)$ is the time-depended Black-Scholes volatility of an option.

Here, the time-depended volatility $\sigma(\tau)$ is related to the annualized volatility σ via a simple formula: $\sigma(\tau) = \sigma\sqrt{\tau}$

Implied rates

Implied $Q(\tau)$ and $R(\tau)$ are computed by dxPrice for each option series and are distributed in TheoPrice event for each option and in Series event for each option series. Both of these values are not necessarily non-negative, because they represent a mix of different factors and correspond to effective dividends and interests experienced by option market makers, while include such factors as cost of carry for both the underlying instrument and for the underlying currency.
Implied simple dividend rate Implied $Q(\tau)$ and interest rate $R(\tau)$ are available via TheoPrice.getDividend and TheoPrice.getInterest or Series.getDividend and Series.getInterest methods correspondingly.

American options

Call-put parity for American options is an inequality of the following form: $C - P \geq U - K$
Implied simple dividend return $Q(\tau)$ and simple interest return $R(\tau)$ are considered to be zero for American options.

4. Hardware acceleration, real-time calculations and analytics

Our latest developments extend calculation engine with Nvidia CUDA GPU Acceleration, Advanced Vector Extensions (AVX) for Intel processors to provide computational power needed to calculate data in real time.

Please contact dxfeed-sales@devexperts.com to learn about custom solutions.